**Interior Angle Sum Theorem**

If a convex polygon has \( n \) sides and \( S \) is the sum of the measures of its interior angles, then

\[
S = 180(n - 2).
\]

**Example:**

\[
\begin{align*}
S &= 180(n - 2) \\
&= 180(5 - 2) \\
&= 180(3) \\
&= 540
\end{align*}
\]
**Example 1** *Interior Angles of Regular Polygons*

**CHEMISTRY** The benzene molecule, $C_6H_6$, consists of six carbon atoms in a regular hexagonal pattern with a hydrogen atom attached to each carbon atom. Find the sum of the measures of the interior angles of the hexagon.

Since the molecule is a convex polygon, we can use the Interior Angle Sum Theorem.

$$S = 180(n - 2) \quad \text{Interior Angle Sum Theorem}$$

$$= 180(6 - 2) \quad n = 6$$

$$= 180(4) \text{ or } 720 \quad \text{Simplify.}$$

The sum of the measures of the interior angles is 720.

The Interior Angle Sum Theorem can also be used to find the number of sides in a regular polygon if you are given the measure of one interior angle.

**Example 2** *Sides of a Polygon*

The measure of an interior angle of a regular polygon is 108. Find the number of sides in the polygon.

Use the Interior Angle Sum Theorem to write an equation to solve for $n$, the number of sides.

$$S = 180(n - 2) \quad \text{Interior Angle Sum Theorem}$$

$$(108)n = 180(n - 2) \quad S = 108n$$

$$108n = 180n - 360 \quad \text{Distributive Property}$$

$$0 = 72n - 360 \quad \text{Subtract } 108n \text{ from each side.}$$

$$360 = 72n \quad \text{Add } 360 \text{ to each side.}$$

$$5 = n \quad \text{Divide each side by } 72.$$

The polygon has 5 sides.

In Example 2, the Interior Angle Sum Theorem was applied to a regular polygon. In Example 3, we will apply this theorem to a quadrilateral that is not a regular polygon.

**Example 3** *Interior Angles*

**ALGEBRA** Find the measure of each interior angle.

Since $n = 4$, the sum of the measures of the interior angles is $180(4 - 2)$ or 360. Write an equation to express the sum of the measures of the interior angles of the polygon.

$$360 = m\angle A + m\angle B + m\angle C + m\angle D \quad \text{Sum of measures of angles}$$

$$360 = x + 2x + 2x + x \quad \text{Substitution}$$

$$360 = 6x \quad \text{Combine like terms.}$$

$$60 = x \quad \text{Divide each side by } 6.$$

Use the value of $x$ to find the measure of each angle.

$$m\angle A = 60, \ m\angle B = 2 \cdot 60 \text{ or } 120, \ m\angle C = 2 \cdot 60 \text{ or } 120, \text{ and } m\angle D = 60.$$
SUM OF MEASURES OF EXTERIOR ANGLES

The Interior Angle Sum Theorem relates the interior angles of a convex polygon to the number of sides. Is there a relationship among the exterior angles of a convex polygon?

Geometry Activity

Sum of the Exterior Angles of a Polygon

Collect Data

- Draw a triangle, a convex quadrilateral, a convex pentagon, a convex hexagon, and a convex heptagon.
- Extend the sides of each polygon to form exactly one exterior angle at each vertex.
- Use a protractor to measure each exterior angle of each polygon and record it on your drawing.

Analyze the Data

1. Copy and complete the table.

<table>
<thead>
<tr>
<th>Polygon</th>
<th>triangle</th>
<th>quadrilateral</th>
<th>pentagon</th>
<th>hexagon</th>
<th>heptagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of exterior angles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sum of measure of exterior angles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What conjecture can you make?

The Geometry Activity suggests Theorem 8.2.

Theorem 8.2

Exterior Angle Sum Theorem

If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360.

Example:

\[ m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360 \]

You will prove Theorem 8.2 in Exercise 42.

Example 4 Exterior Angles

Find the measures of an exterior angle and an interior angle of convex regular octagon \( ABCDEFGH \).

At each vertex, extend a side to form one exterior angle. The sum of the measures of the exterior angles is 360.

A convex regular octagon has 8 congruent exterior angles.

\[ 8n = 360 \quad n = \text{measure of each exterior angle} \]

\[ n = 45 \quad \text{Divide each side by 8.} \]

The measure of each exterior angle is 45. Since each exterior angle and its corresponding interior angle form a linear pair, the measure of the interior angle is 180 - 45 or 135.
1. **Explain** why the Interior Angle Sum Theorem and the Exterior Angle Sum Theorem only apply to convex polygons.

2. **Determine** whether the Interior Angle Sum Theorem and the Exterior Angle Sum Theorem apply to polygons that are not regular. Explain.

3. **OPEN ENDED** Draw a regular convex polygon and a convex polygon that is not regular with the same number of sides. Find the sum of the interior angles for each.

**Guided Practice**

Find the sum of the measures of the interior angles of each convex polygon.

4. pentagon

5. dodecagon

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

6. 60

7. 90

**ALGEBRA** Find the measure of each interior angle.

8. \[ \frac{(3x - 4)\degree}{x} + \frac{x\degree}{(3x - 4)\degree} \]

9. \[ \frac{2x\degree}{(9x + 30)\degree} + \frac{(9x + 30)\degree}{2x\degree} \]

Find the measures of an exterior angle and an interior angle given the number of sides of each regular polygon.

10. 6

11. 18

**Application**

12. AQUARIUMS The regular polygon at the right is the base of a fish tank. Find the sum of the measures of the interior angles of the pentagon.

**Practice and Apply**

Find the sum of the measures of the interior angles of each convex polygon.

13. 32-gon

14. 18-gon

15. 19-gon

16. 27-gon

17. 4y-gon

18. 2x-gon

19. GARDENING Carlotta is designing a garden for her backyard. She wants a flower bed shaped like a regular octagon. Find the sum of the measures of the interior angles of the octagon.

20. GAZEBOS A company is building regular hexagonal gazebos. Find the sum of the measures of the interior angles of the hexagon.

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

21. 140

22. 170

23. 160

24. 165

25. 157\frac{1}{2}

26. 176\frac{2}{5}
**ALGEBRA** Find the measure of each interior angle using the given information.

27. \( \triangle MNPQ \) with \( m \angle M = 10x \) and \( m \angle N = 20x \)

29. parallelogram \( MNPQ \) with \( m \angle M = 10x \) and \( m \angle N = 20x \)

30. isosceles trapezoid \( TWYZ \) with \( \angle Z \equiv \angle Y, m \angle Z = 30x \), \( \angle T \equiv \angle W \), and \( m \angle T = 20x \)

31. decagon in which the measures of the interior angles are \( x + 5 \), \( x + 10 \), \( x + 20 \), \( x + 30 \), \( x + 35 \), \( x + 40 \), \( x + 60 \), \( x + 70 \), \( x + 80 \), and \( x + 90 \)

32. polygon \( ABCDE \) with \( m \angle A = 6x \), \( m \angle B = 4x + 13 \), \( m \angle C = x + 9 \), \( m \angle D = 2x - 8 \), and \( m \angle E = 4x - 1 \)

33. quadrilateral in which the measures of the angles are consecutive multiples of \( x \)

34. quadrilateral in which the measure of each consecutive angle increases by 10

Find the measures of each exterior angle and each interior angle for each regular polygon.

35. decagon

36. hexagon

37. nonagon

38. octagon

Find the measures of an interior angle and an exterior angle given the number of sides of each regular polygon. Round to the nearest tenth if necessary.

39. 11

40. 7

41. 12

42. **PROOF** Use algebra to prove the Exterior Angle Sum Theorem.

43. **ARCHITECTURE** The Pentagon building in Washington, D.C., was designed to resemble a regular pentagon. Find the measure of an interior angle and an exterior angle of the courtyard.

44. **ARCHITECTURE** Compare the dome to the architectural elements on each side of the dome. Are the interior and exterior angles the same? Find the measures of the interior and exterior angles.

45. **CRITICAL THINKING** Two formulas can be used to find the measure of an interior angle of a regular polygon: \( s = \frac{180(n - 2)}{n} \) and \( s = 180 - \frac{360}{n} \). Show that these are equivalent.
46. **Writing in Math**  
Answer the question that was posed at the beginning of the lesson.

How does a scallop shell illustrate the angles of polygons?  
Include the following in your answer:  
• explain how triangles are related to the Interior Angle Sum Theorem, and  
• describe how to find the measure of an exterior angle of a polygon.

47. A regular pentagon and a square share a mutual vertex X.  
The sides XY and XZ are sides of a third regular polygon with a vertex at X. How many sides does this polygon have?  
- A 19  
- B 20  
- C 28  
- D 32

48. **GRID IN** If $6x + 3y = 48$ and $\frac{9y}{2x} = 9$, then $x =$ ?

49. In $\triangle ABC$, given the lengths of the sides, find the measure of the given angle to the nearest tenth.  
(Lesson 7-7)  
49. a = 6, b = 9, c = 11; $m\angle C$  
50. a = 15.5, b = 23.6, c = 25.1; $m\angle B$  
51. a = 47, b = 53, c = 56; $m\angle A$  
52. a = 12, b = 14, c = 16; $m\angle C$

Solve each $\triangle FGH$ described below. Round angle measures to the nearest degree and side measures to the nearest tenth.  
(Lesson 7-6)  
53. $f = 15$, $g = 17$, $m\angle F = 54$  
54. $m\angle F = 47$, $m\angle H = 78$, $g = 31$  
55. $m\angle G = 56$, $m\angle H = 67$, $g = 63$  
56. $g = 30.7$, $h = 32.4$, $m\angle G = 65$

57. **Proof**  
Write a two-column proof.  
(Lesson 4-5)  
Given: $\overline{JK} \parallel \overline{LM}$  
Prove: $\triangle JKL \cong \triangle MLK$

State the transversal that forms each pair of angles. Then identify the special name for the angle pair.  
(Lesson 3-1)  
58. $\angle 3$ and $\angle 11$  
59. $\angle 6$ and $\angle 7$  
60. $\angle 8$ and $\angle 10$  
61. $\angle 12$ and $\angle 16$

**Getting Ready for the Next Lesson**  
**PREREQUISITE SKILL**  
In the figure, $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. Name all pairs of angles for each type indicated.  
(To review angles formed by parallel lines and a transversal, see Lesson 3-1.)  
62. consecutive interior angles  
63. alternate interior angles  
64. corresponding angles  
65. alternate exterior angles